

(37)

Exponential Decay

An exponential decay occurs when a quantity decreases by the same rate in a period of time.

$$y = a(1-r)^t \rightarrow \text{time}$$

↓ ↓ ↓
total initial rate
amount value

ex) The population in a town is decreasing at a rate of 3% per year. In 2000, there were 1,700 people. Write an exponential function to model this situation. Then find the population in 2012.

$$y = 1700(1 - 0.03)^{12}$$
$$y = 1180$$

ex) A fish population in a local stream is depreciating at a rate of 4.25% per year. The original population was 48,000. Write an exponential function to model this situation. Then find the population after 7 years.

$$y = 48,000(1 - 0.0425)^7$$
$$y = 35,417$$

A common application of exponential decay is half-life.

The Half-life of a substance is the time it takes for one-half of the substance to decay into another substance.

$$\begin{array}{ccc} A = P(0.5)^t & \rightarrow & \text{amount of half-lives} \\ \downarrow & \downarrow & \text{in a given time.} \\ \text{total} & \text{initial} & \\ \text{amount} & \text{amount} & \end{array}$$

ex) Astatine-218 has a half-life of 2 seconds.
Find the amount left from a 500 gram sample of Astatine-218 after 10 seconds.

$$t = \frac{10}{2} = 5$$

$$A = 500(0.5)^5$$

$$A = 15.625 \text{ g}$$

ex) Astatine-218 has a half-life of 2 seconds.
Find the amount left from a 500 gram sample of Astatine-218 after 1 minute.

$$t = \frac{60}{2} = 30$$

$$A = 500(0.5)^{30}$$

$$A = 0.000000466 \text{ g}$$

ex) Cesium-137 has a half-life of 30 years. Find the amount left from a 100 milligram sample of Cesium-137 after 180 years.

$$t = 180/30 = 6$$

$$A = 100(0.5)^6$$

$$A = 1.5625 \text{ mg}$$